temperature was changed. The thermocouple was used for measuring temperatures above 77°K and the resistance thermometer below. The resistance-temperature characteristics of the germanium thermometer were taken from the Honeywell calibration tables for the thermometer. Over the temperature range below 77°K, the temperature was measured to better than 0.5 deg and was controlled to at least one degree over the time required for a measurement.

The cryostat assembly used for determinations of the elastic moduli using the resonance method consisted of a specimen chamber, heater, and support placed in an insulated Dewar. The specimen chamber was made of copper and placed directly against the upper end of a Kanthol wound heater. Both the specimen chamber and the heater were supported by a brass fitting set on the bottom of a stainless steel Dewar-flask (Hoffman model S/N 210-22). The assembly was immersed in a liquid-nitrogen bath to make the measurement at 77°K. Temperature of the specimen chamber was controlled by a variac and readings were taken by setting the variac and allowing a sufficient time for thermal equilibrium to be approached closely. The time interval allowed for this purpose varied from 30 min to about two hours. Over the temperature range 77°–300°K, a copper–constantan thermocouple (which had been calibrated previously against an NBS standard) was used to measure the specimen temperature. A frosting of water vapor on the specimen surface was prevented by a constant flow of precooled N_2 gas. The specimen temperature was kept constant within 1 deg during the time required for measurements.

Measurements of resonant frequencies as a function of temperature above 300°K were made in a Kanthol wound tube-furnace connected to a variac. The upper temperature limit of the furnace was about 1330°K. The specimen temperature was measured with a platinum–platinum plus 10% rhodium thermocouple which had been calibrated earlier against an NBS standard, and was recorded with an accuracy better than ±3 deg.

2.4. Hydrostatic Pressure System

The equipment used for the generation of hydrostatic pressure was constructed by the Harwood Engineering Company, Walpole, Mass. The unit was designed for the production of hydrostatic pressures to about 14 kbar. Argon gas was the pressure medium. The pressure was measured with a manganin cell in connection with a Foxboro Dynalog recorder. The temperature of the sample was kept constant by the circulation of warm water outside of the entire pressure cell. The temperature of a copper plate which was in direct contact with one of the specimen faces was measured with a chromel–alumel thermocouple. With another thermocouple, the temperature was measured on the opposite face of the specimen to guard against the presence of thermal gradients within the specimen. In each determination, the frequency was recorded only after ascertaining the system had reached thermal equilibrium; usually, a wait of 15–20 min after changing pressure was sufficient for this purpose.

3. EXPERIMENTAL RESULTS

3.1. Variation of the Isotropic Elastic Moduli with Pressure

The quantity of interest in our pressure experiments is the first derivative of an elastic modulus M with respect to hydrostatic pressure evaluated at zero-pressure, $(dM/dp)_{p=0}$. This derivative is an isothermal one. For a modulus M, we have

$$M=4\rho l^2 f^2, \tag{3.1}$$

where f is the corrected repetition frequency. Taking logarithms and differentiating both sides with respect to pressure,

$$d(\ln M)/dp=d(\ln\rho)/dp+2[d(\ln l)/dp]+d(\ln f^2)/dp. \tag{3.2}$$

Since $(d\ln l/dp)=(d\ln V/dp)/3$ in our case, and $(d\ln\rho/dp)=-(d\ln V/dp)$, where V is volume, and after evaluating the derivatives at $p=0$, we have

$$[d(\ln M)/dp]_{p=0}=(1/3B^T)_{p=0}+[(d/dp)(f/f_0)^2]_{p=0} \tag{3.3}$$

or

$$(dM/dp)_{p=0}=(M/3B^T)_{p=0}+[M\cdot(d/dp)(f/f_0)^2]_{p=0}. \tag{3.4}$$

B^T is the isothermal bulk modulus and it is related to the adiabatic bulk modulus B^s by $B^T=B^s(1+T\beta\gamma_G)^{-1}$, where β is the coefficient of volume expansion, γ_G is Grüneisen's ratio, and T is temperature in °K.

Table I summarizes the isotropic elastic properties of the specimen at 298°K along with the corresponding quantities evaluated for nonporous polycrystalline aluminas. Table II is a listing of the relative change in the linear dimension of the specimen, the ratio of the frequency squared, and density as a function of hydrostatic pressure up to 10 kbar. The values of (l/l_0) were found according to Cook's approximation scheme.[20] The corresponding density values were calculated from $\rho=\rho_0(l_0/l)^3$, where ρ_0 is the initial density of the specimen. The sound velocities at a given pressure p were then calculated according to $v_j=v_{j(0)}(f_j/f_{j(0)})(l/l_0)$, where the subscript (0) denotes the quantity at the unstrained condition; the isotropic elastic moduli were then calculated in the usual way.

The first pressure derivatives of sound velocities at 298°K were found as follows: $(dv_l/dp)=5.35$ and $(dv_t/dp)=2.20$ in units of 10^{-3} (km/sec)/kbar. The

[20] R. K. Cook, J. Acoust. Soc. Am. 29, 445 (1957).

TABLE I. Density, sound velocities, and isotropic elastic moduli of the alumina specimen and the corresponding quantities evaluated at the single-crystal density-point (at 298°K).

Quantity	Measured values	Values[b] corrected for porosity
Density (g/cm^3)	3.974	3.986
Longitudinal velocity (km/sec)	10.845(±0.003)[a]	10.889
Transverse velocity (km/sec)	6.377(±0.002)[a]	6.398
Adiabatic longitudinal modulus ($\times 10^{11}$ dyn/cm^2)	46.739	47.262
Shear modulus ($\times 10^{11}$ dyn/cm^2)	16.160	16.316
Adiabatic bulk modulus ($\times 10^{11}$ dyn/cm^2)	25.192	25.507
Isothermal bulk modulus ($\times 10^{11}$ dyn/cm^2)	25.033	25.346

[a] These variations in sound velocities represent the observed variations in the velocities when the direction of wave propagation was changed. Since these variations were less than the expected experimental error, the specimen was considered elastically isotropic (see text for the description).
[b] Values obtained from Ref. 8.

corresponding derivatives of the isotropic elastic moduli were: from Eq. (3.4), $(dL^s/dp)=6.51$, $(dG/dp)=1.77$, and $(dB^s/dp)=4.16$ calculated from the former, whereas using Cook's method, 6.5, 1.8 and 4.2, respectively. These values may be compared with the cor-

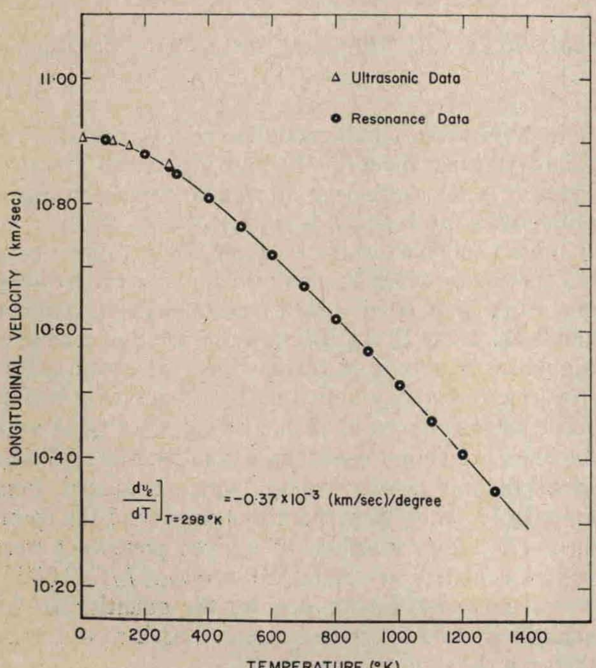

FIG. 1. Temperature dependence of the longitudinal sound velocity determined on a polycrystalline alumina specimen of $\rho_0=3.974$ g/cm^3. Data points consist of two kinds: one, velocities resulting directly from the ultrasonic method and the other, velocities calculated from the density and measured isotropic elastic moduli resulting from the bar-resonant method.

TABLE II. The relative change in the linear dimension, ratio of the frequency squared, and density as a function of hydrostatic pressure (at 298°K).

Pressure (kbar)	(l/l_0)	Density (g/cm^3)	$(f_l/f_{l(0)})^2$	$(f_t/f_{t(0)})^2$
0.001	1.000000	3.974	1.00000	1.00000
1.0	0.999867	3.976	1.00127	1.00097
2.0	0.999735	3.977	1.00252	1.00193
3.0	0.999603	3.979	1.00378	1.00289
4.0	0.999470	3.980	1.00504	1.00385
5.0	0.999338	3.982	1.00630	1.00481
6.0	0.999206	3.984	1.00757	1.00577
7.0	0.999076	3.985	1.00882	1.00673
8.0	0.998943	3.987	1.01008	1.00769
9.0	0.998811	3.988	1.01135	1.00865
10.0	0.998682	3.990	1.01260	1.00960

responding quantities obtained on a Lucalox alumina specimen by Schreiber and Anderson.[12] Their values were: $(dL^s/dp)=6.34$, $(dG/dp)=1.75$, and $(dB^s/dp)=3.98$ at 298°K.

3.2. Variation of the Isotropic Elastic Parameters with Temperature

Figures 1 and 2 are plots of sound velocities as a function of temperature. The data points below 300°K consist of two kinds: one, velocities resulting directly

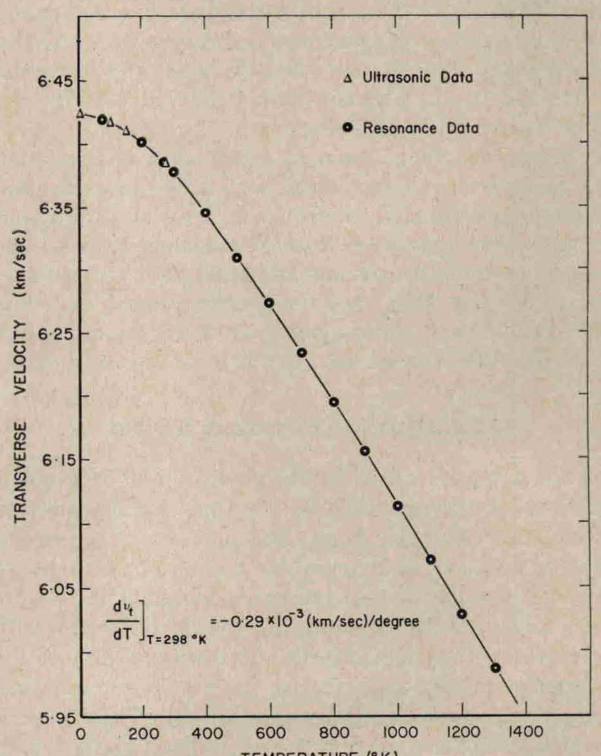

FIG. 2. Temperature dependence of the transverse sound velocity determined on a polycrystalline-alumina specimen of $\rho_0=3.974$ g/cm^3. Data points consist of two kinds: one, velocities resulting directly from the ultrasonic method and the other, velocities calculated from the density and measured shear modulus resulting from the bar-resonant method.